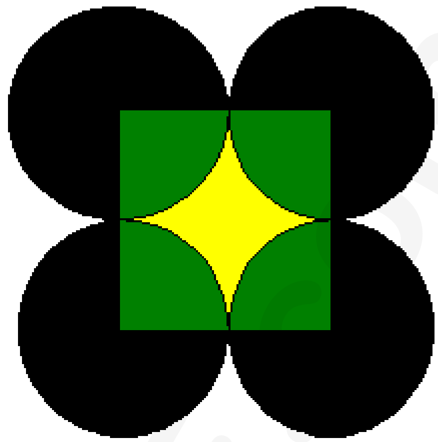
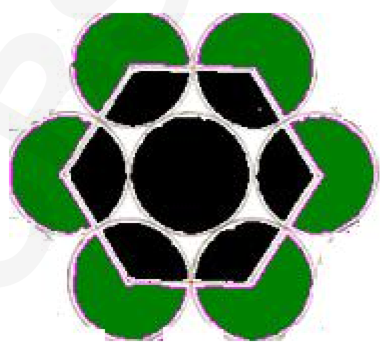
Efficiency in Packing



Square Packing



Hexagonal Packing

Efficiency in packing

Objective

To investigate the efficiency of packing of objects of different shapes in a cuboid box.

Efficiency is the percentage of box space occupied by the objects.

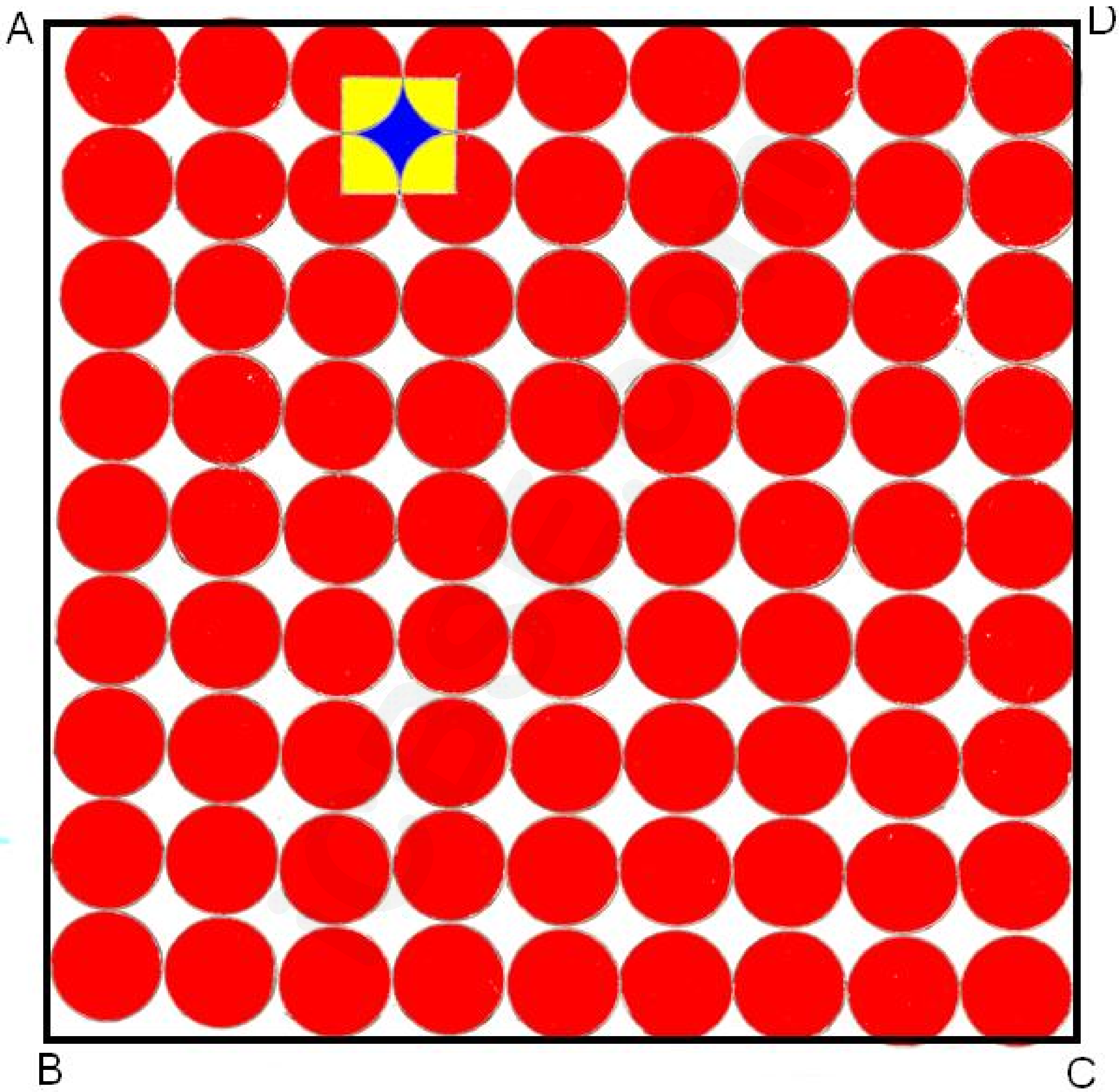
Description

1. **Took a certain number of cylindrical tins and packed them in a cuboid container.**
   - **(a) For illustration I took 81 tins.**
   - **(b) Second time I took 64 tins.**
   - **(c) Third time I took 49 tins.**

2. **The cylindrical tins can be placed in two different ways These are**
   - **(a) Square packing**
   - **(b) Hexagonal packing**

3. **I wished to study which packing out of two is more efficient.**

4. **To understand the difference between the two packing I have drawn figures on the left side pages.**


A
D
B
C

Example 1 (81 tins)

Calculation

Case 1
Square packing

Each base circle is circumscribed by a square.
Area of one circle = πR^2

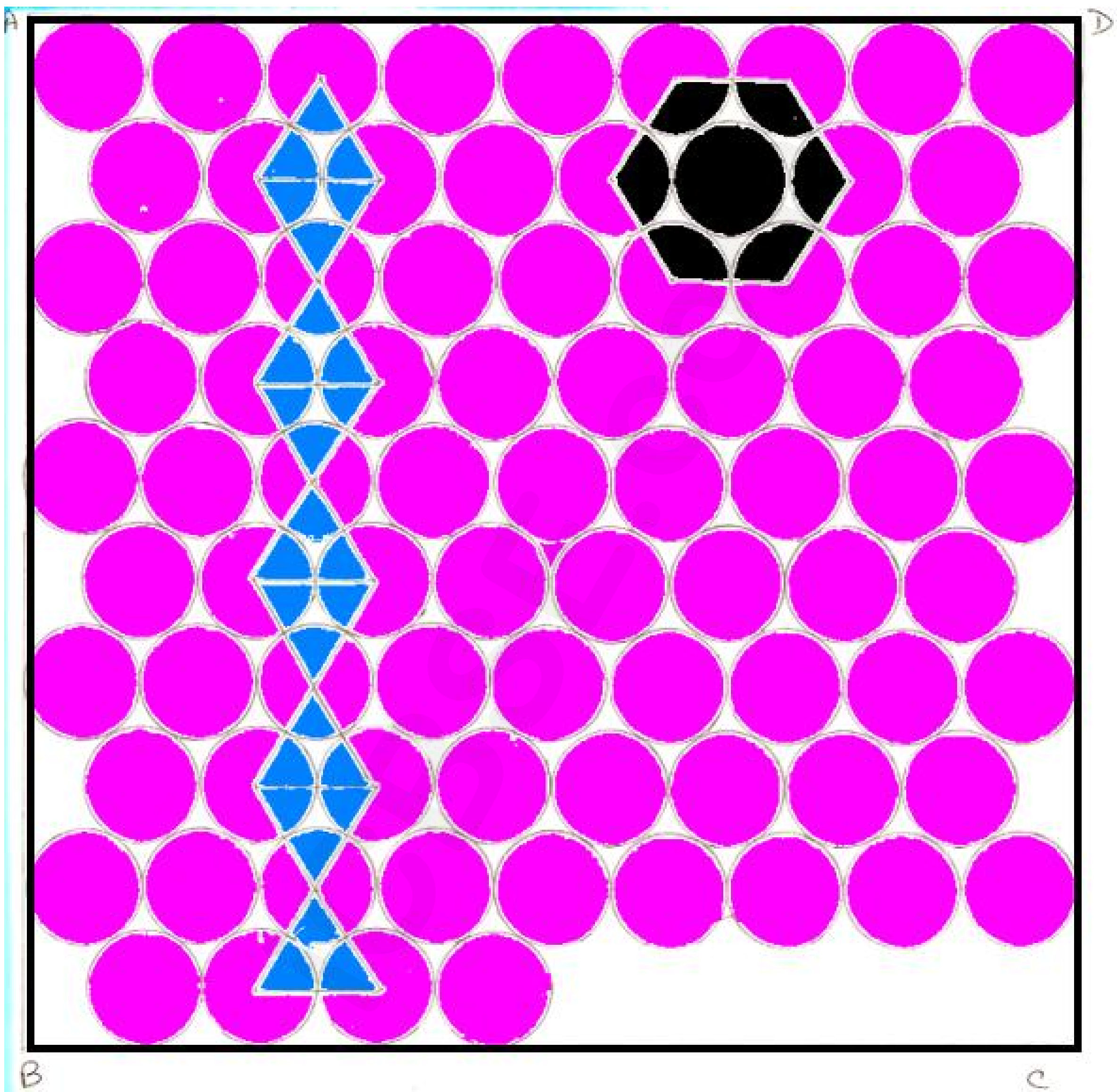
Area of square = $4 R^2$

Area of circle / area of square = $\pi R^2 / 4 R^2$

This ratio will be evidently the same as the cross section of all the tins to the total base area.

Percentage efficiency = $\pi / 4 \times 100$
= 78.5 %

Therefore the efficiency in case of square packing is 78.5%


A
D
B
C

Case 2
Hexagonal packing

Here we determine the sides of the base of the container in terms of the radius of the cylindrical tin.

One side of the rectangular base i.e. BC = 18 × R.

To determine the other side, AB = 2 × R + 9 × *h*, where *h* is the altitude of the equilateral triangle formed by joining the centres of three adjacent circles.

$h = 2R \sin 60°$

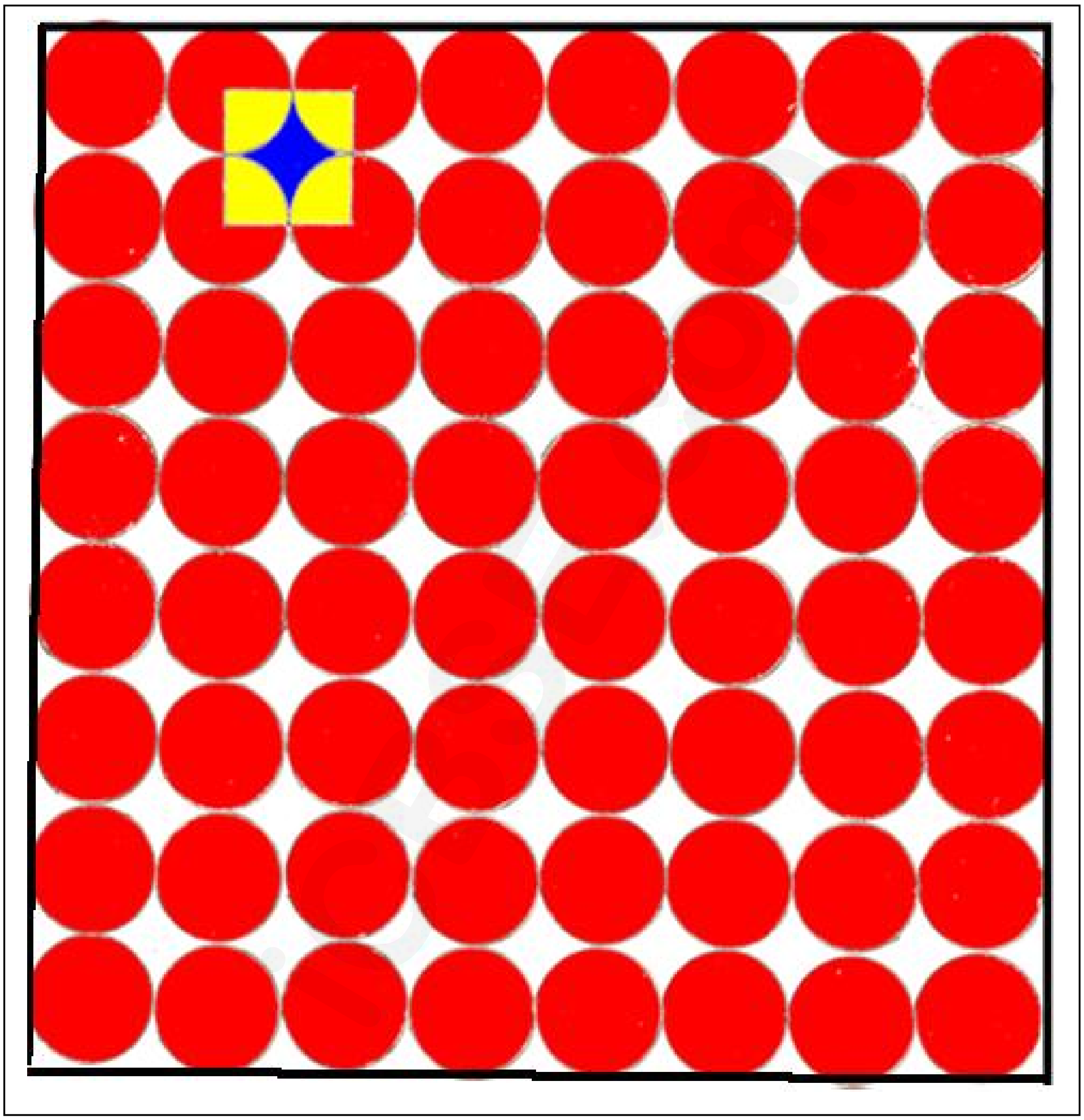
$AB = 2R + 18 R \sin 60°$

$\sin 60° = \sqrt{3} / 2$

Now $AB = 2R + 18R \times \sqrt{3} / 2$
$= (2 + 9\sqrt{3}) R$

Area of ABCD = $18R \times (2 + 9\sqrt{3}) R$
$= 18R^2 (2 + 9\sqrt{3})$

Percentage efficiency = $81\pi R^2 \times 100 / 18R^2 (2 + 9\sqrt{3})$
$= 80.3$ %



Example 2 (64 tins)

Calculation

Case 1
Square packing

Each base circle is circumscribed by a square.
Area of one circle = πR^2

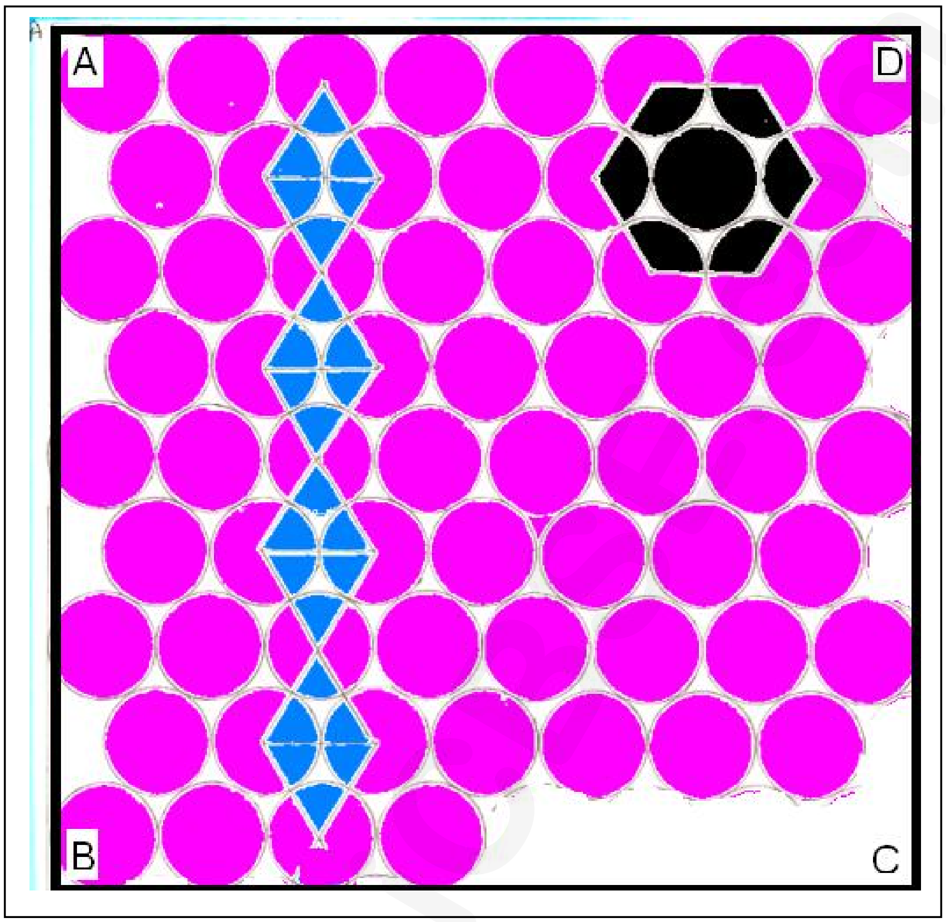
Area of square = $4 R^2$

Area of circle / area of square = $\pi R^2 / 4 R^2$

This ratio will be evidently the same as the cross section of all the tins to the total base area.

Percentage efficiency = $\pi / 4 \times 100$
= 78.5 %

Therefore the efficiency in case of square packing is 78.5%


A
D
B
C

Case 2
Hexagonal packing

Here we determine the sides of the base of the container in terms of the radius of the cylindrical tin.

One side of the rectangular base i.e. BC = 16 × R.

To determine the other side, AB = 2 × R + 8 × *h*, where *h* is the altitude of the equilateral triangle formed by joining the centres of three adjacent circles.

$h = 2R \sin 60°$

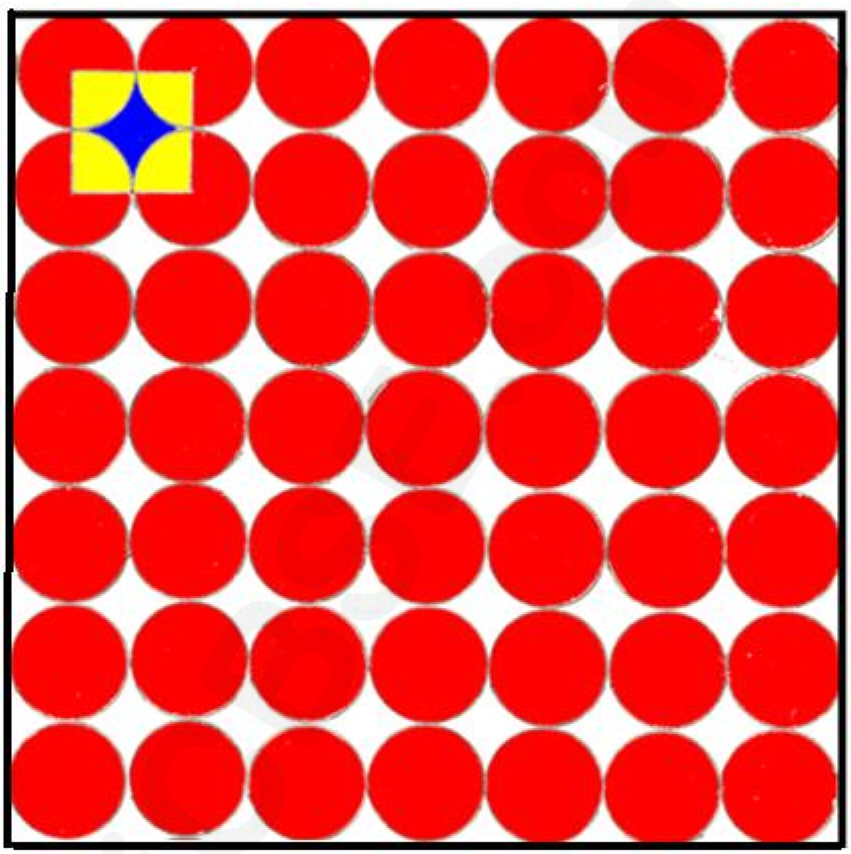
$AB = 2R + 16 R \sin 60°$

but $\sin 60° = \sqrt{3} / 2$

Now $AB = 2R + 16R \times \sqrt{3} / 2$
$= 2R + 8R\sqrt{3}$
$= (1 + 4\sqrt{3})\, 2R$

Area of ABCD $= 16R \times (1 + 4\sqrt{3})\, 2R$
$= 32R^2 (1 + 4\sqrt{3})$

Percentage efficiency $= 64\pi R^2 \times 100 / 32R^2 (1 + 4\sqrt{3})$
$= 79.3$ %



Example 3 (49 tins)

Calculation

Case 1
Square packing

Each base circle is circumscribed by a square.
Area of one circle = πR^2

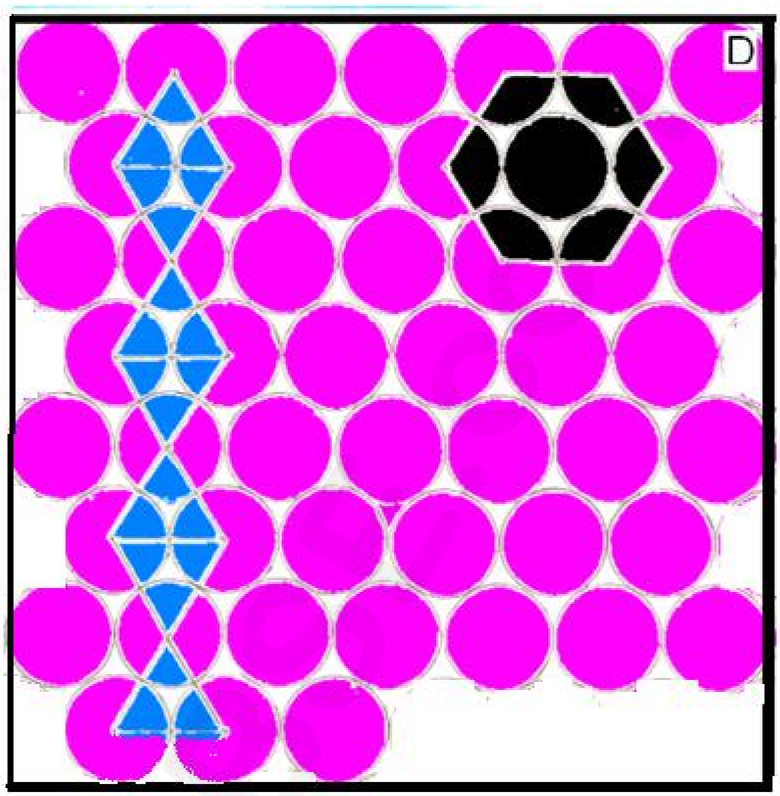
Area of square = $4 R^2$

Area of circle / area of square = $\pi R^2 / 4 R^2$

This ratio will be evidently the same as the cross section of all the tins to the total base area.

Percentage efficiency = $\pi / 4 \times 100$
= 78.5 %

Therefore the efficiency in case of square packing is 78.5%


D

Case 2
Hexagonal packing

Here we determine the sides of the base of the container in terms of the radius of the cylindrical tin.

One side of the rectangular base i.e. BC = 14 × R.

To determine the other side, AB = 2 × R + 7 × *h*, where *h* is the altitude of the equilateral triangle formed by joining the centres of three adjacent circles.

$h = 2R \sin 60°$

$AB = 2R + 14\,R \sin 60°$

but $\sin 60° = \sqrt{3} / 2$

Now $AB = 2R + 14R \times \sqrt{3} / 2$
$= 2R + 7R\sqrt{3}$
$= (2 + 7\sqrt{3})\,R$

Area of ABCD $= 14R \times (2 + 7\sqrt{3})\,R$
$= 14R^2\,(2 + 7\sqrt{3})$

Percentage efficiency $= 49\pi R^2 \times 100 / 14R^2\,(2 + 7\sqrt{3})$
$= 77.96\,\%$

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Remarks

1. In the calculations here the number of tins was fixed and the cuboid dimensions are variable.

A similar exercise may be done with fixed cuboid dimensions and variable number of tins.

2. We can also determine the efficiency for packing of spheres in a cuboid.

Volume of sphere = $4/3\ \pi R^3$

Volume of cube = $8R^3$

Percentage efficiency = $4\ \pi\ R^3\ /\ 3 \times 8\ R^3$

= $\pi\ /\ 6$

= 52 %

Note

When 81 and 64 tins were taken **Hexagonal packing** was more efficient but in case of 49 tins **Square packing** was more efficient.