Displacement and rotation of a geometrical figure

Objective:

To study between different points of a geometrical figure when it is displacement and/or rotated. Enhance familiarity with co-ordinate geometry.

Description:

1. A cut out of a geometrical figure such as a triangle is made and placed on a rectangular sheet of paper marked with X and Y-axis.
2. The co-ordinates of the vertices of the triangle and its centroid are noted.
3. The triangular cut out is displaced (along x-axis, along y-axis or along any other direction.)
4. The new co-ordinates of the vertices and the centroid are noted again.
5. The procedure is repeated, this time by rotating the triangle as well as displacing it. The new co-ordinate of vertices and centroid are noted again.

6. Using the distance formula, distance between the vertices of the triangle are obtained for the triangle in original position and in various displaced and noted positions.
7. Using the new co-ordinates of the vertices and the centroids, students will obtain the ratio in which the centroid divides the medians for various displaced and rotated positions of the triangles.

Result:

Students will verify that under any displacement and rotation of a triangle the displacement between verticals remain unchanged, also the centroid divides the medians in ratio 2:1 in all cases.

Conclusion:

In this project the students verify (by the method of co-ordinate geometry) What is obvious geometrically, named that the length of a triangle do not change when the triangle is displaced or rotated. This project will develop their familiarity with co-ordinates, distance formula and section formula of co-ordinate geometry.

When the triangle cut out is kept at Position (1 Quadrant) as shown in Fig: 4.1 following fortification are made:

Vertices of triangle are A (3, 6), B (1, 3), C (6, 3).

By midpoint formula. D is midpoint of BC and its co-ordinates are calculated as shown below-

Midpoint = $\left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$

$= \left[\frac{1+6}{2}, \frac{3+3}{2}\right]$

$= \left[\frac{7}{2}, \frac{6}{2}\right]$

= D (3.5, 3)

The centroid calculated by formula is x co-ordinate of centroid is

$\frac{3+1+6}{3} = \frac{10}{3}$ = 3.3

y coordinate of centroid is $\frac{6+3+3}{3} = \frac{12}{3} = 4$

I Position Graph

Co-ordinates of centroid are G (3.3, 4)

Distance AG $= \sqrt{(3.3-3)^2 + (4-6)^2}$

$= \sqrt{(.3)^2 + (-2)^2}$

$= \sqrt{0.09+4}$

$= \sqrt{4.09}$

$\therefore GD = 2.02$

Distance GD $= \sqrt{(3.3-3.5)^2 + (4-3)^2}$

$= \sqrt{0.04+1}$

$= \sqrt{0.04}$

$\therefore GD = 1.01$

Ratio $= \frac{AG}{GD} = \frac{2.02}{1.01}$

AG: GD = 2:1

When the triangle cut out is kept at position 11 (11quadrant) as shown in fig. 4.2 following notifications are made :

Vertices of triangles are A (-4, 6), B (-6, 3) and C (-1, 3)

By midpoint formula D is midpoint of BC and its co-ordinates are calculated as shown below.

$$\text{Midpoint} = \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$$

$$= \left[\frac{-6+(-1)}{2}, \frac{3+3}{2}\right]$$

$$= \left[\frac{-7}{2}, \frac{6}{2}\right]$$

$$\therefore D = (-3.5, 3)$$

The centroid calculated by formula is x co-ordinate of centroid is

$$\frac{-4+-6+-1}{3} = \frac{-11}{3} = -3.6$$

II Position Graph

Y co-ordinates of centroid is $\frac{6+3+3}{3} = \frac{12}{3} = 4$

Co-ordinates of centroid are G (-3.6, 4)

Distance AG = $\sqrt{(-3.6+4)^2+(4-6)^2}$

= $\sqrt{(0.4)^2+(-2)^2}$

= $\sqrt{0.16+4}$

= $\sqrt{4.16}$

∴ AG = 2.03

Distance DG = $\sqrt{(-3.6+3.5)^2+(3-4)^2}$

= $\sqrt{0.01+1}$

= $\sqrt{1.01}$

∴ DG = 1.00

Ratio

$\frac{AG}{DG} = \frac{2.03}{1.00} = \frac{2}{1}$

AG : DG = 2 : 1

When the triangle cut out is kept at position III (III quadrant) as shown in Fig. 4.3 following notification are made:

Vertices of triangle are A (-3, -6), B (-1, -3) and C (-6, -3)

By midpoint formula D is midpoint of BC and its co-ordinates are calculated as shown below:

$$\text{Midpoint} = \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$$

$$= \left[\frac{-1+(-6)}{2}, \frac{-3-3}{2}\right]$$

$$= \left[\frac{-7}{2}, \frac{-6}{2}\right]$$

$$= [-3.5, -3]$$

The centroid calculated by formula is x co-ordinate of centroid is

$$\frac{-3+-1+-6}{3} = \frac{-10}{3} = \text{-3.3}$$

y co-ordinates of centroid is $\frac{-6+(-3)+(-3)}{3} = \frac{-12}{3} = -4$

III Position

Co-ordinates of centroid are G (-3.3, -4)

Distance AG = $\sqrt{(-3.3+3)^2+(-4+6)^2}^2$

= $\sqrt{(-0.3)^2+(2)^2}^2$

= $\sqrt{0.09+4}$

= $\sqrt{4.09}$

AG = 2. 03

Distance DG = $\sqrt{(-3.3+3.5)^2+(-4+3)^2}^2$

= $\sqrt{0.04+1}$

= $\sqrt{1.04}$

DG = 1. 01

Ratio = $\frac{AG}{DG} = \frac{2.03}{1.01} = \frac{2}{1}$

∴ AG : DG = 2 : 1

When the triangle cut out is kept at position 1V (1V quadrant) as shown in Fig. 4.4 following notifications are made:

Vertices of triangle are A (4, -6), B (1, -3) and C (6, -3)

By midpoint formula D is midpoint of BC and its Co-ordinates are calculated as shown below:

Midpoint

$$= \left[\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right]$$

$$= \left[\frac{1+6}{2}, \frac{-3-3}{2}\right]$$

$$= \left[\frac{7}{2}, \frac{-6}{2}\right]$$

$$= \text{D } [3.5, -3]$$

The centroid calculated by formula is x Co-ordinate of centroid is

$\frac{4+1+6}{3} = \frac{11}{3}$ = 3. 6

y Co-ordinate of centroid is $\frac{-6-3-3}{3} = \frac{-12}{4}$ = -4

IV Position Graph

Co-ordinate of centroid are G (3.6, -4)

Distance AG = $\sqrt{(3.6 - 4)^2 + (4 - 6)^2}$

= $\sqrt{(-0.4)^2 + (-2)^2}$

= $\sqrt{0.16 + 4}$

= $\sqrt{4.16}$

AG = 2. 03

Distance DG = $\sqrt{(3.6 - 3.5)^2 + (-4 + 3)^2}$

= $\sqrt{0.01 + 1}$

= $\sqrt{1.01}$

DG = 1.00

Ratio = $\frac{AG}{DG} = \frac{2.03}{1.00} = \frac{2}{1}$

AG : DG = 2 : 1